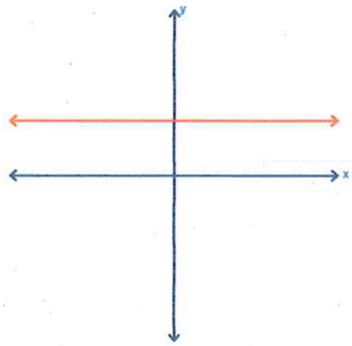
## Common "Parent" Functions

The eight graphs shown in the figures below represent the most commonly used functions in algebra. Familiarity with the basic characteristics of these simple graphs will help you analyze the shapes of more complicated graphs – in particular, graphs obtained from these common graphs by the rigid and non-rigid transformations that are studied in the next section.

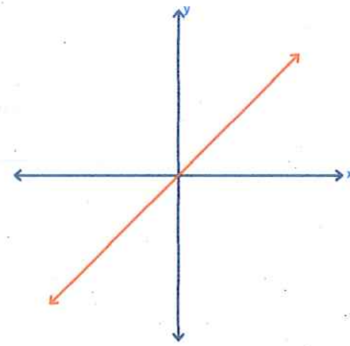
Constant

$f(x) = c$



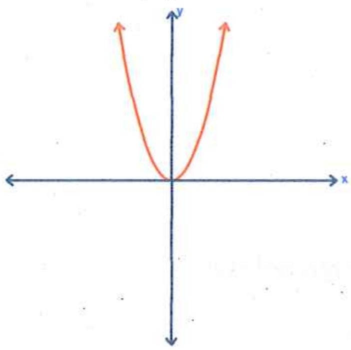
Identity (or Linear)

$f(x) = x$



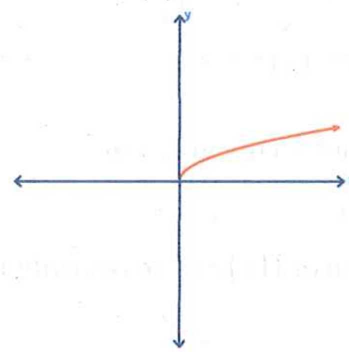
Quadratic

$f(x) = x^2$



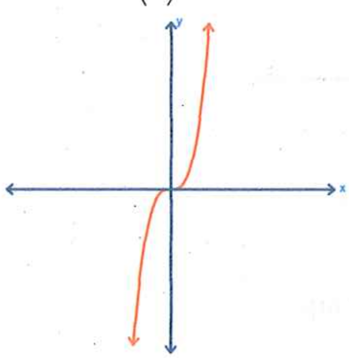
Square Root

$f(x) = \sqrt{x}$



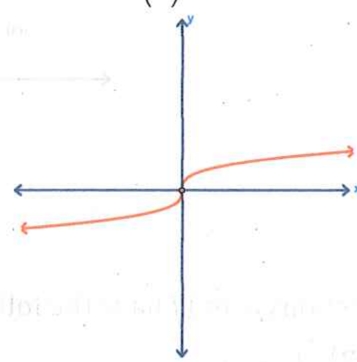
Cubic

$f(x) = x^3$



Cubic Root

$f(x) = \sqrt[3]{x}$

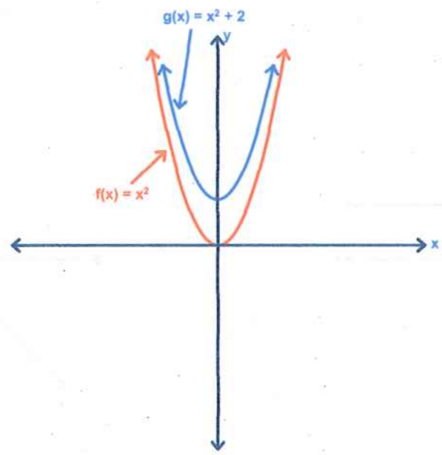


## Shifting Graphs

Many functions have graphs that are simple transformations of the parent graphs we talked about in the previous section. For example, you can obtain the graph of

$$h(x)=x^2+2$$

by shifting the graph of $f(x)=x^2$ *upward* two units, as show below:




In function notation, *h* and *f* are related as follows:

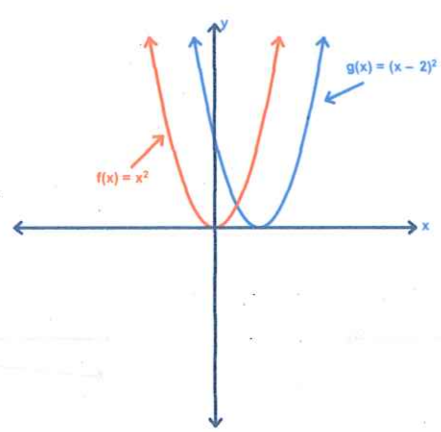
$$h(x)=f(x)+2$$

which means $h(x)=x^2+2$

Similarly, you can obtain the graph of

$$g(x)=(x-2)^2$$

by shifting the graph of $f(x)=x^2$ *to the right* two units, as shown below:




In this case, the functions *g* and *f* have the following relationship

$$g(x)=f(x-2)=(x-2)^2$$

These shifts are called *rigid transformations* and can be summarized as shown below:

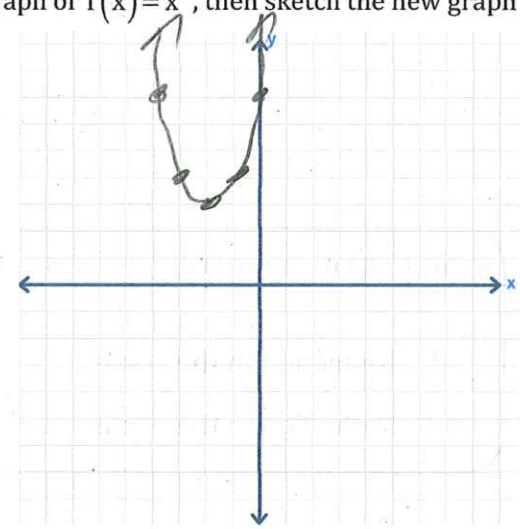
Let c be a positive real number. Vertical and horizontal shifts in the graph of $y=f(x)$, are represented as follows:

| | | *Transformation rule:* |
|---|---|---|
| 1. Vertical shift *c* units *upward:* | $h(x)=f(x)+c$ | $(x,y)\rightarrow(x,y+c)$ |
| 2. Vertical shift *c* units *downward:* | $h(x)=f(x)-c$ | $(x,y)\rightarrow(x,y-c)$ |
| 3. Horizontal shift *c* units to the *right:* | $h(x)=f(x-c)$ | $(x,y)\rightarrow(x+c,y)$ |
| 4. Horizontal shift *c* units to the *left:* | $h(x)=f(x+c)$ | $(x,y)\rightarrow(x-c,y)$ |

**Example:**

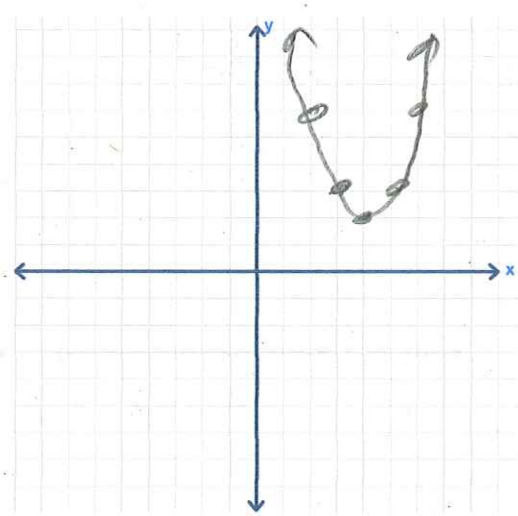
Describe in words how the graph $y=(x+2)^2+3$ will be transformed from the graph of $f(x)=x^2$, then sketch the new graph

shift left 2
shift up 3




**Try this:** Describe, in words, how the graph $y=(x-4)^2+2$ will be transformed from the graph of $f(x)=x^2$, then sketch the graph

shift right 4
shift up 2



## Reflecting Graphs

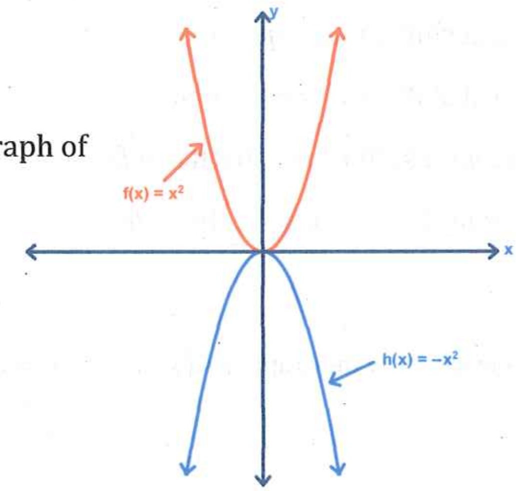
The second common type of transformation is a *reflection*.

If you consider the *x*-axis to be a mirror, the graph of

$$h(x) = -x^2$$

is the **reflection over the x-axis** (or mirror image) of the graph of

$$f(x) = x^2$$

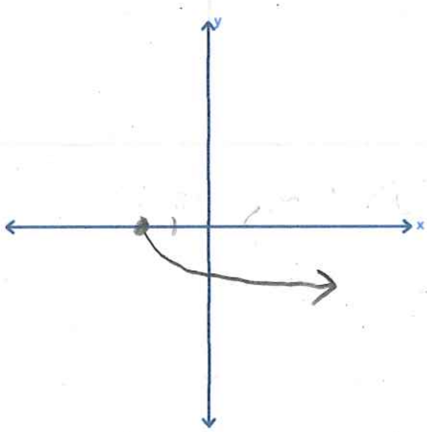



*Reflections* in the coordinate axes of the graph of $y = f(x)$ are represented as follows:

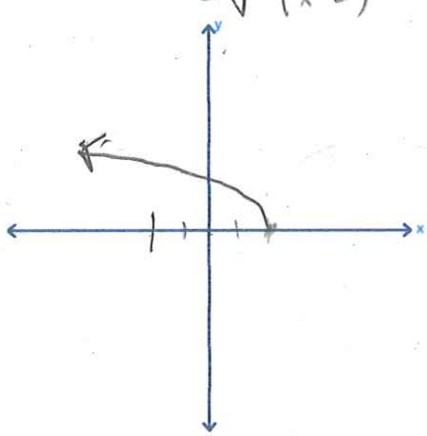
| | | *Transformation rule:* |
|---|---|---|
| 1. Reflection in the *x*-axis: | $h(x) = -f(x)$ | $(x,y) \to (x,-y)$ |
| 2. Reflection in the *y*-axis: | $h(x) = f(-x)$ | $(x,y) \to (-x,y)$ |

**Example:**
Sketch the following graphs without using your calculator:

$y = -\sqrt{x+2}$



$y = \sqrt{-x+2}$
$= \sqrt{-(x-2)}$



$y = \sqrt{-(x+2)}$